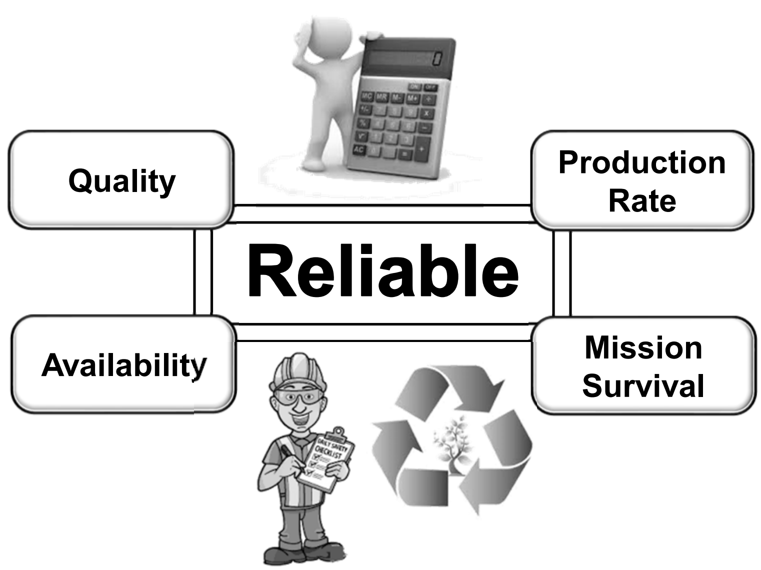
04.1 Reliability analysis

RA: 2 dimensions

1

1



2

2

Agenda

1. Reliability
2. Reliability analysis is counting
3. The real meaning of the six RCM curves
4. Conditional Probability of Failure
5. conditional probability, reliability, and failure rate
6. Random failure
7. Weibull Analysis and PM Optimization Workshop

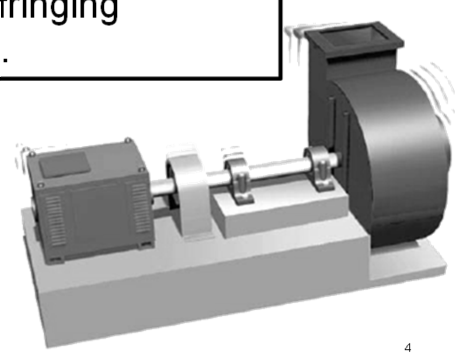
 3

3

Reliability definition

Probability that a new item will perform its stated function until a stated age t.

The achievement of a desired production rate, quality, availability, mission survivability, at lowest cost, safely, and without infringing environmental norms.



 4

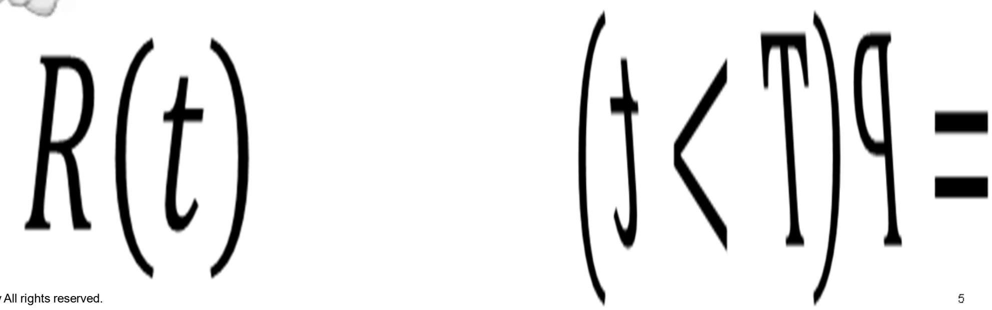
4

Reliability definition

Probability that a new item will perform its stated function until a stated age t.

$$R(t) = P(T > t)$$

$$T\ is\ t\ \ e\ age\ at\ failure$$



 5

5

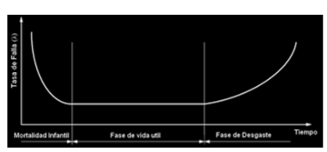
Conditional failure probability H(t) and Failure rate h(t)

Conditional failure probability H(t): The probability of failing in upcoming age interval Δt from the current age t.

$$H(t) = \frac{R(t) \quad R(t + \quad t)}{R(t)}$$

Failure rate h(t): The limit of $\frac{H(t)}{t} as \quad t \rightarrow 0$

$$(t) = \lim_{t \to 0} \frac{R(t) \quad R(t + \quad t)}{tR(t)}$$

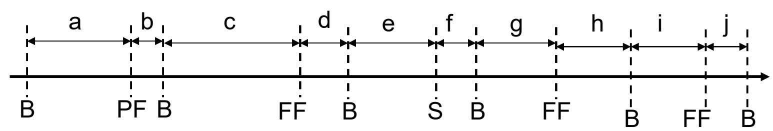


e.g.Bathtub curve

 6

6

MTTF, MTTR, MTBF



"reliability" $MTTF \approx \dfrac{a + c + g + i}{4}$ Average life*

"maintainability" $MTTR = \dfrac{b+d+f+h+j}{5}$ Average repair time

$$MTBF = MTTF + MTTR$$

*Ignoring suspensions

7

7

Financial analysis is counting money

8

8

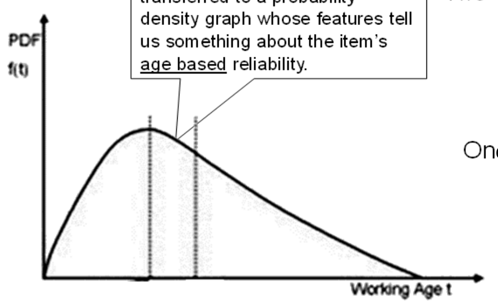
Reliability analysis is counting instances of unreliability (failure)

One

Two

Three

The count in each <u>age group</u> is *transferred to a probability* density graph whose features tell us something about the item's <u>age based</u> reliability.



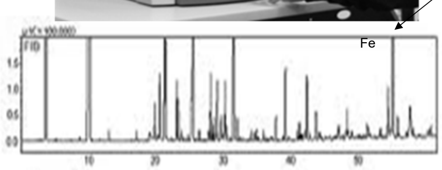
One

Two

9

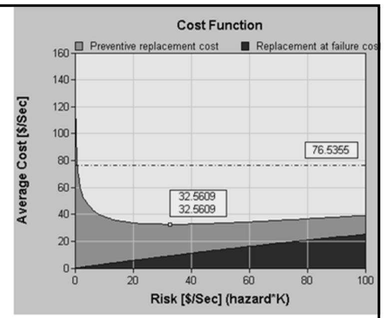
CBM Predictive Reliability Analysis

Is counting the number of times a monitored (large) value precedes a certain failure mode event.



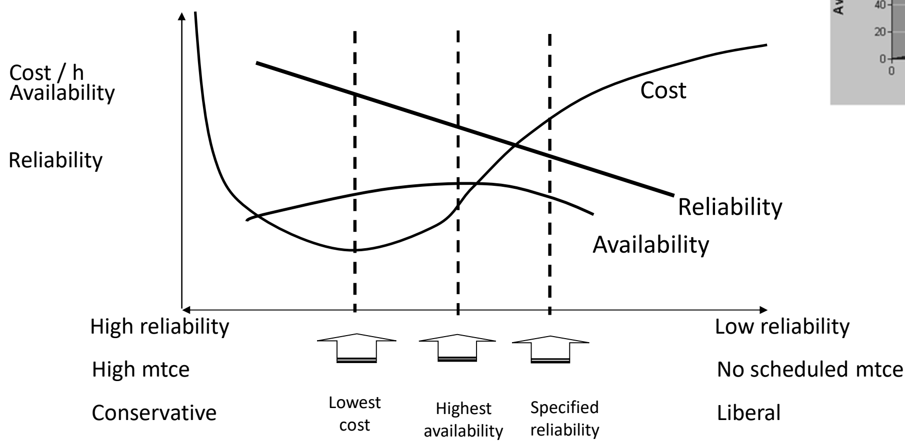
10

10



What do we mean by “optimal”?

What level of maintenance (policy) do we want?



 11

11

Conditional failure probability → decision power

If you knew that the conditional probability of failure you could…

1. Pre-empt the consequences of failure
2. Avoid waste of time and resources
3. Reduce downtime
4. Increase reliability

 12

12

What is the conditional probability?

Card experiment:

Pick a card.
Pick another card.
(without replacing the first card)

What is the probability that

A. the first card chosen is a queen

<u>and</u>

B. the second card chosen is a jack?

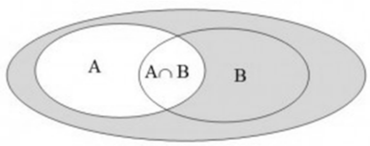
Event B depends on Event A

Conditional Probability

P(queen on first pick) = P(A) = 4/52

P(jack on 2nd pick given queen on 1st pick)= P(B|A) = 4/51

The probability of both events, P(A and B) = P(A) ∩ P(B) = (4/52)(4/51)=4/663



P(A∩B) = P(A) · P(B|A)

$$P(B|A) = \frac{P(A) \cap P(B)}{P(A)}$$

13

13

What is the conditional probability of failure?

Suppose the two dependent events A and B were:

	Event	Description
A.	X>t	the item survives to time t
B.	t≤X≤t+t,	the item fails in the interval between t and t+Δt

Obviously, event B depends on Event A.

Recall from previous slide that: $P(B|A) = \frac{P(A) \cap P(B)}{P(A)}$

Subsituting the definitions of A and B from the table:

$$H(t) = P(\underbrace{t \le X \le t + \;t}_{B} | \underbrace{X > t}_{A}) = \frac{\overbrace{P(t \le X \le t + \;t)}^{B} \cap \overbrace{P(X > t)}^{A}}{\underbrace{P(X > t)}_{A}}$$

14

14

Conditional probability of failure is a *special case* of conditional probability

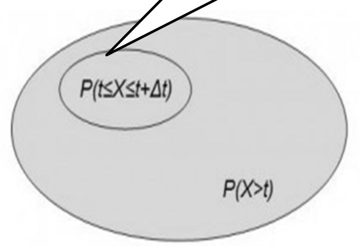
From previous slide:

$$H(t) = P(t \leq X \leq t + \quad t | X > t) = \frac{P(t \leq X \leq t + \quad t) \cap P(X > t)}{P(X > t)}$$

So the intersection of the two probabilities, the numerator is simply P(t≤X≤t+Δt).

Therefore:

But P(t≤X≤t+Δt) lies entirely inside the probability space of P(X>t).

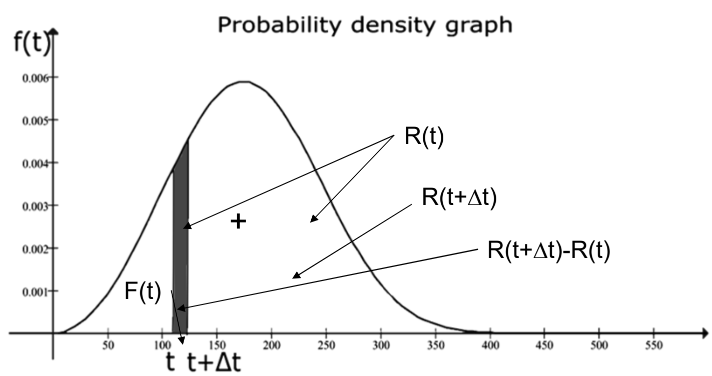


$$H(t) = \frac{P(t \leq X \leq t + \quad t)}{P(X > t)}$$

15

15

Conditional failure probability and reliability,



$$H(t) = \frac{P(t \leq X \leq t + \quad t)}{P(X > t)}$$

R(t)-R(t+Δt)

R(t)

Therefore: $H(t) = \frac{R(t) \quad R(t + \quad t)}{R(t)}$

16

16

What about the failure rate?

From previous slide we have:

$$H(t) = \frac{R(t) \quad R(t+\quad t)}{R(t)}$$

N&H poined out the failure rate h(t) is the limit of the ratio H(t)/Δt as Δt→0:

$$(t) = \lim_{t \to 0} \frac{R(t) \quad R(t+\quad t)}{R(t)\ \ t} = \frac{1}{R(t)}\left(\frac{dR(t)}{dt}\right)$$

$F(t)$ =*1-R(t)*

$$\frac{dF(t)}{dt} = \quad \frac{dR(t)}{dt}$$

$f(t)$

substituting

$$(t) = \frac{f(t)}{R(t)}$$

17

17

"Random" failure – is it really random?

It depends on how you define "random".
It is "random" in this sense. Consider throws of a di. Define "failure" as throwing a "6". Then:

1. Conditional probability of failure would be constant (1/6)
   - Regardless of how many previous throws, the probability of failure on the next throw is still 1/6. I.E. failure is independent of age, age being the number of throws
2. *Reliability* (probability of not throwing a 6) decays exponentially*.

*Exponential: something changes at set intervals by a constant factor.

A bank savings account grows exponentially by the factor (1+ interest rate).
When the factor < 1, we have with "exponential decay".

18

18

Example: Your car

Context:

- You drive normally.
- You replace tires when treads go below safety depth.

Intuitively, we would agree that you're no more likely to have a punctured tire in any one year than in any other. <u>The conditional failure probability H is constant.</u>

Assume that the probability of having a flat tire is 25% in any one year.

That is the conditional failure probability H(t) is a constant 25%

What is the reliability (with respect to tire failure) R1 at the moment you drive the new care off the dealer's lot?

R1=100%

19

19

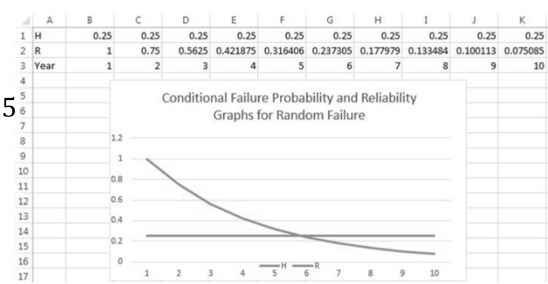
What is the Reliability at beginning of year 2?

$$H(t) = \frac{R(t) \quad R(t + \quad t)}{R(t)} \quad = \frac{R1 \quad R2}{R1} \quad = 25\%$$

R2 = R1 - .25$\times R1$

R2 = 1 - .25$\times 1 = 0.75$

R3 = R2 - .25$\times R2 = 0.5625$



	A	B	C	D	E	F	G	H	I	J	K
1	H	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
2	R	1	0.75	0.5625	0.421875	0.316406	0.237305	0.177979	0.133484	0.100113	0.075085
3	Year	1	2	3	4	5	6	7	8	9	10

The Reliability graph says that there is a 7.5% chance (row 2 col K) that you will drive for 9 years without a puncture.

20

20

4.1.1 Quiz 1 Reliability Analysis in 2D

https://forms.gle/zJJc7KHD8nQXLBYE9

Which of the following statements about reliability is correct? Reliability: * 1 point

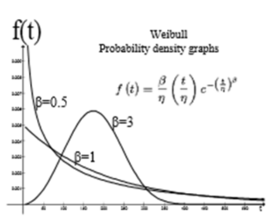
- ○ is the probability that a new item will survive to specified age.
- ○ is the probability that a new item will perform satisfactorily and safely over a specified time.
- ○ is the probability that an item will survive to a specified age or time.
- ○ can refer to an equipment, component, part, or failure mode.
- ○ of an item is often approximated by its MTTF.
- ○ All of the above.
- ○ None of the above.

 21

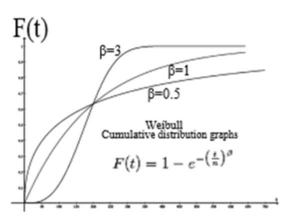
21

Summary: Reliability concepts illustrated with Weibull graphs.

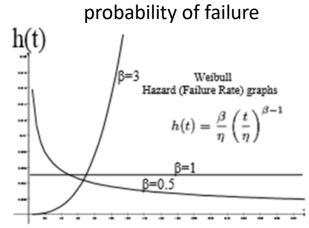
f(t): Probability density



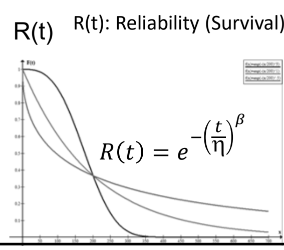
F(t): Cumulative failure probability



h(t): Failure rate, hazard, H(t): Conditional probability of failure



R(t): Reliability (Survival)



 22

22

Reliability Summary
Different ways of representing the same probabilty distribution

F(t) Cumulative distribution function(CDF). The area under curve f(t) from 0 to t. $F(t)=\int_0^\infty f(t)dt$

f(t) Probability density $f(t)=\dfrac{dF(t)}{dt}$

R(t) Survival function. (also called "Reliability function".) R(t) = 1-F(t)

h(t) failure rate. (aka hazard.) h(t) = f(t)/R(t)

H(t) Conditional probabilty of failure H(t)= (R(t)-R(t+L))/R(t).

Probabily that the ítem fails in interval [t to t+L] given that it has survived to t. Same shape as h(t)

MTTF Expected life. (aka mean time to failure, expected failure age) $MTTF=\int_0^\infty tf(t)dt$

23

23

4.1.1 Quiz 2 Reliability Analysis in 2D

https://forms.gle/SHVjze7dFZXQDk6A8

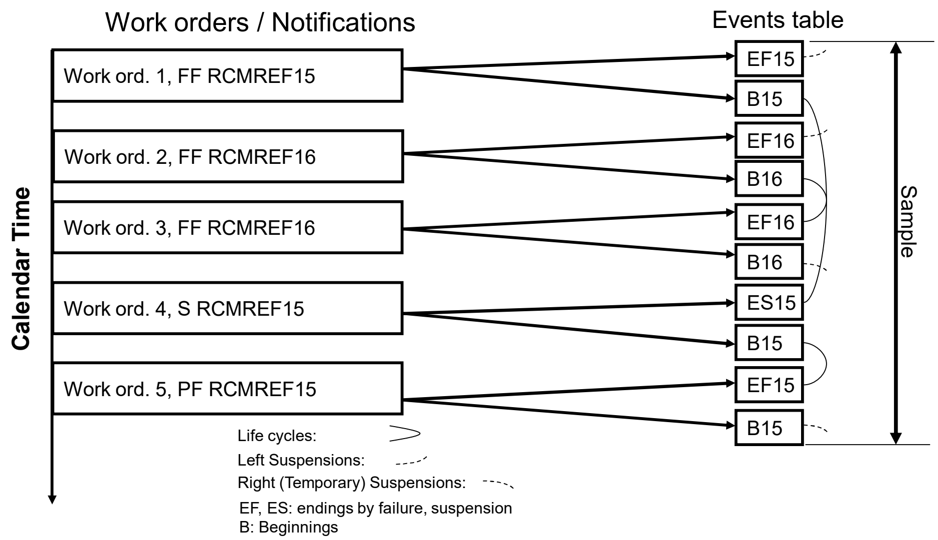
Random failure means: * 1 point

- The probability of an item's failure during the upcoming interval, if it has survived to start of that interval, is constant.
- The item's reliability decays exponentially.
- If you flipped a coin 5 times and got tails each time, the probability of tails on the next flip is still 50%
- As long as your tires' tread depth is > the safety limit the probability of a blowout in any year is constant.
- All of the above.
- None of the above.

24

24

4.1.2 What is a sample?



25

25